10.2 HW Solutions to #'s 53 & 69

53.) $\begin{bmatrix} -1 & 1 & 1 & -1 \\ -1 & 2 & -3 & -4 \\ 3 & -2 & -7 & 0 \end{bmatrix}$ $R_2 = r_1 - r_2$ $\rightarrow \begin{bmatrix} -1 & 1 & 1 & -1 \\ 0 & -1 & 4 & 3 \\ 3 & -2 & -7 & 0 \end{bmatrix}$ $R_3 = 3r_1 - r_3$ $\begin{bmatrix} -1 & 1 & 1 & -1 \\ 0 & -1 & 4 & 3 \\ 0 & 1 & -4 & -3 \end{bmatrix}$

$R_1 = -1 \cdot r_1$ $\begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & -1 & 4 & 3 \\ 0 & 1 & -4 & -3 \end{bmatrix}$ $R_3 = r_2 + r_3$ $\begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & -1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 = -r_2$ $\begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & -4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x - y - z = 1$

$y - 4z = -3 \rightarrow y = 4z - 3$

$x = y + z + 1 \rightarrow x = (4z - 3) + z + 1 \Rightarrow$

$x = 5z - 2$

$y = 4z - 3$

$z =$ Any real #

69.) $\begin{bmatrix} 2 & 3 & -1 & 3 \\ 1 & -1 & -1 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 3 & 5 \end{bmatrix}$ $r_1 \Longleftrightarrow r_4$ $\rightarrow \begin{bmatrix} 1 & 1 & 3 & 5 \\ 1 & -1 & -1 & 0 \\ -1 & 1 & 1 & 0 \\ 2 & 3 & -1 & 3 \end{bmatrix}$ $R_2 = r_1 - r_2$ $\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 2 & 4 & 5 \\ -1 & 1 & 1 & 0 \\ 2 & 3 & -1 & 3 \end{bmatrix}$

$R_3 = r_1 + r_3$ $\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 2 & 4 & 5 \\ 0 & 2 & 4 & 5 \\ 2 & 3 & -1 & 3 \end{bmatrix}$ $R_4 = -2r_1 + r_4$ $\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 2 & 4 & 5 \\ 0 & 2 & 4 & 5 \\ 0 & 1 & -7 & -7 \end{bmatrix}$ $R_2 = \frac{1}{2} r_2$ $\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 1 & 2 & 5/2 \\ 0 & 2 & 4 & 5 \\ 0 & 1 & -7 & -7 \end{bmatrix}$ $R_3 = -2r_4 + r_3$ $\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 1 & 2 & 5/2 \\ 0 & 0 & 18 & 19 \\ 0 & 1 & -7 & -7 \end{bmatrix}$

$R_4 = -1r_2 + r_4$ $\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 1 & 2 & 5/2 \\ 0 & 0 & 18 & 19 \\ 0 & 0 & -9 & -\frac{19}{2} \end{bmatrix}$ $R_3 = \frac{1}{18} r_3$ $\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 1 & 2 & 5/2 \\ 0 & 0 & 1 & 19/18 \\ 0 & 0 & -9 & -\frac{19}{2} \end{bmatrix}$ $R_4 = 9r_3 + r_4$ $\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 1 & 2 & 5/2 \\ 0 & 0 & 1 & 19/18 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x + \left(\frac{7}{18}\right) + 3\left(\frac{19}{18}\right) = 5 \Rightarrow \boxed{x = \frac{13}{9}}$

$y + 2\left(\frac{19}{18}\right) = \frac{5}{2} \Rightarrow \boxed{y = \frac{7}{18}}$

$\boxed{z = \frac{19}{18}}$